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PROBLEMS LEADING TO THE CONCEPT OF A MATRIX AND METHODS OF THEIR SOLUTION

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Abstract
The article is devoted to teaching the topic of matrices and actions on them, a branch of linear algebra in higher mathematics. In the traditional approach, this topic is usually presented solely from a mathematical point of view, with the solution of examples to demonstrate concepts. In this article, unlike others, for a better understanding by students, it shows what a matrix is, what tasks lead to the concept of a matrix, and what advantages solving these problems using matrices provides compared to other methods.

Keywords: Table, determinant, matrix, matrix multiplication and addition, matrix method and Gauss method.

Introduction
Students studying at higher technical educational institutions often ask the question: "Why do we need to study certain topics in higher mathematics, in particular matrices?" Therefore, it is explained to students why it is necessary to study each topic (the main question) in higher mathematics, in particular the topic of matrices and operations on them. Which will be useful in studying problems encountered in fields such as engineering, agriculture, and economics, and how these problems can be solved using mathematical models. They are clearly explained through the knowledge that will be gained on the topic of matrices. The advantages of solving these problems using matrices in comparison with other methods are shown below.

LITERATURE ANALYSIS: In existing textbooks on higher mathematics [1,2,4,6], most attention is paid to theoretical knowledge, and only in rare cases, brief information about the practical application of this knowledge is given at the end of the topic. Therefore, in the traditional teaching of higher mathematics in universities, methods for solving abstract problems that model natural processes and practical problems in a quantitative form are first studied. Later, these mathematical methods are used in teaching special subjects.

METHODOLOGY: The article presents theoretical and practical problems leading to the concept of a matrix. Several examples are considered in which systems of linear algebraic equations are solved using matrices, which simplifies calculations [7,8,9,11].

DISCUSSION AND RESULTS: Consider the following practical problems leading to the concept of a matrix. To begin with, let's consider a problem that answers the question: "Why do we need matrices (determinants)?" [7]. Task: The amount of cotton harvested by 15 pickers on farm plots before and after lunch was recorded in separate tables.

day / n/a	1	2	3	4	5	6	7	8	9	10
1	45	50	53	61	38	49	70	48	56	59
2	55	60	52	51	78	59	75	55	76	69
3	44	55	55	62	59	59	61	56	57	63
4	55	52	55	41	42	45	61	52	52	50
5	45	50	52	61	35	59	50	45	52	52
6	46	50	65	70	45	56	70	65	50	55
7	45	52	75	60	20	45	52	62	58	55
8	45	61	52	63	65	65	56	52	72	0
9	75	50	55	71	65	58	65	85	50	55
10	62	62	85	65	65	52	41	75	10	64
11	63	75	71	71	75	52	56	51	45	52
12	52	63	52	65	75	45	45	54	52	85
13	60	51	52	65	70	85	52	53	56	52
14	61	45	52	0	56	69	68	52	54	58
15	62	75	65	56	58	59	69	75	96	54

day / n/a	1	2	3	4	5	6	7	8	9	10
1	55	41	42	45	61	52	52	50	52	57
2	52	61	35	59	50	45	52	52	45	52
3	65	70	45	56	70	65	50	55	65	52
4	75	60	20	45	52	62	58	55	62	50
5	50	53	61	38	49	70	48	56	59	58
6	60	52	51	78	59	75	55	76	69	72
7	55	55	62	59	59	61	56	57	63	50
8	52	55	41	42	45	61	52	52	50	10
9	50	52	61	35	59	50	45	52	52	45
10	50	65	70	45	56	70	65	50	55	52
11	52	75	60	20	45	52	62	58	55	56
12	61	52	63	65	65	56	52	72	0	54
13	50	55	71	65	58	65	85	50	55	96
14	62	85	65	65	52	41	75	10	64	64
15	75	71	71	75	52	56	51	45	52	52

To determine the amount of cotton they collected per day and for 10 days, it is necessary to add up these tables. In order to calculate their wages, multiply them by a certain amount (the amount paid for 1 kilogram of harvested cotton).

In addition, when preparing statistical reports, when studying the coefficients of a system of equations that express a mathematical model of a physical process, it is necessary to work with tables.

Namely, perform the operations of addition, multiplication by a number, and multiplication of these tables. Therefore, it is important to study such tables. To perform operations on such tables, the concepts of matrix and determinant are used. These quantities are studied in detail in mathematics courses. The article presents the tasks that lead to the concepts of matrix and determinant, as well as the operations performed with them.

To begin with, we will study the method that will be necessary when analyzing the results of experimental tests to create new varieties in agriculture. In this case, performing operations on matrices simplifies calculations [3,5,7].

Task: A farmer randomly selected 50 bushes from a plot of one hectare of his plantation to determine the yield of cotton. He counted the number of fully formed boxes on these bushes. The results were recorded in the following matrix (table).

10	12	11	8	9	11	8	8	6	9
9	10	9	7	11	10	9	8	8	13
11	12	10	7	9	9	9	8	10	10
7	8	10	9	9	8	10	9	10	11
13	10	8	10	9	9	8	10	11	8

One bush of Bukhoro-102 cotton, planted in a field, produces an average of 4-5 grams of cotton. Therefore, we assume that about 4.5 grams of cotton comes out of one bush. In order to find the mass of cotton that forms on each bush, multiply the above matrix by 4.5. Thus, the weight of cotton harvested from n=50 selected bushes will be calculated as follows (in grams):

45.0	54.0	49.5	36.0	40.5	49.5	36.0	36.0	27.0	40.5
40.5	45.0	40.5	31.5	49.5	45.0	40.5	36.0	36.0	58.5
49.5	54.0	45.0	31.5	40.5	40.5	40.5	36.0	45.0	45.0
31.5	36.0	45.0	40.5	40.5	36.0	45.0	40.5	45.0	49.5
58.5	45.0	36.0	45.0	40.5	40.5	36.0	45.0	49.5	36.0.

The average weight of cotton produced from each bush is calculated as follows:

$$\bar{x} = \frac{1}{50}(27 \cdot 1 + 31{,}5 \cdot 3 + 36 \cdot 11 + 40.5 \cdot 13 + 45 \cdot 12 + 49{,}5 \cdot 6 +$$

$$+ 54 \cdot 2 + 58.5 \cdot 2) = \frac{2111}{50} = 42.22 gr.$$

If we take into account that approximately 110,000 bushes grow on one hectare, then the expected yield per hectare will be calculated as follows:

M = 110 000$\bar{x}$ = 110 000 · 42.22 = 4622,2kg ≈46,2 centner.

For students studying economics and accounting, the following problem can be given, which leads to the concept of a matrix [3,5,7].

Task. The salary of 50 workers working in the joint venture for the year was as follows in the form of matrix A (calculated per 1,000,000 soums):

133,6	130,2	130,4	131,6	135,0	132,1	132,2	130,7	129,6	129,8
136,8	137,0	130,2	131,6	132,0	132,1	134,0	134,2	130,4	130,6
133,0	132,6	131,3	131,8	134,0	130,8	129,0	131,0	128,1	128,0
131,6	134,2	135,4	136,0	131,3	132,6	129,2	130,0	136,0	137,0
132,4	132,6	130,0	134,2	132,1	133,1	133,0	133,2	134,1	128,2.

During the year, in connection with the holidays, these workers were paid a bonus equal to 20 times the basic estimated salary (375,000 × 20 = 7,500,000 soums), and advanced and active employees of public work were additionally awarded an additional monthly salary. This is represented as the following matrix B:

18,6	7.5	7,5	7,5	18,8	18,5	18,5	7,5	7,5	7,5
20,2	20,2	7,5	7,5	18,5	18,5	18,6	18,6	7,5	7,5
18,5	18,6	7,5	7,5	18,6	7,5	7,5	7,5	7,5	7,5
7,5	18,6	18,7	18,8	7,5	7,5	7,5	7,5	18,8	18,9
7,5	7,5	7,5	18,6	7,5	18,5	18,5	18,5	18,6	7,5.

In order to find out how much money the employees of the joint venture received in a year, you need to find the sum of these two matrices A and B by adding the corresponding elements.

152,2	137,5	137,9	139,1	153,5	150,6	150,7	138,2	137,1	137,3
153,2	157,2	137,9	139,1	150,5	150.6	152,6	152,8	137,9	138,1
151,5	151,6	138,8	139,3	152,6	138,3	136,5	138,5	135,6	135,5
139.1	152,8	154,1	154,8	138,8	140,1	136,7	137,5	154,8	155,9
139.9	140.1	137.5	152,8	139,6	151,6	151,5	151,7	152,7	135,7

Usually, such reports are written using long records, where calculations are performed separately for each employee. As you can see, such calculations can be easily performed using matrices, encoding (numbering) each employee and using the matrix addition operation on a computer.

Many theoretical and practical problems lead not to a single equation with one unknown, but to a system of several equations with several unknowns. The system of equations can consist of both linear and nonlinear algebraic equations, as well as linear and nonlinear differential equations.

Using the example below, we will show how the use of matrices in solving a system of linear algebraic equations simplifies calculations [2,7,8]:

Task: To solve a system of equations $\begin{cases} x+y+2z=-4 \\ 4x+y+4z=-3 \\ 2x-y+2z=3 \end{cases}$.

1) Let's solve this system using matrices. To do this, we find the determinant of the matrix A, made up of coefficients in front of the unknowns.

$$A=\begin{pmatrix} 1 & 1 & 2 \\ 4 & 1 & 4 \\ 2 & -1 & 2 \end{pmatrix}, \qquad |A|=\begin{vmatrix} 1 & 1 & 2 \\ 4 & 1 & 4 \\ 2 & -1 & 2 \end{vmatrix}=-6$$

By calculating the algebraic complements of the determinant of the system, we find the inverse matrix.

$A^{-1}=\frac{1}{-6}\begin{pmatrix} 6 & -4 & 2 \\ 0 & -2 & 4 \\ -6 & 3 & -3 \end{pmatrix}$. We will find the solution of the equation using the formula

$$X=A^{-1}B=-\frac{1}{6}\begin{pmatrix} 6 & -4 & 2 \\ 0 & -2 & 4 \\ -6 & 3 & -3 \end{pmatrix}\cdot\begin{pmatrix} -4 \\ -3 \\ 3 \end{pmatrix}=-\frac{1}{6}\begin{pmatrix} -24+12+6 \\ 0+6+12 \\ 24-9-9 \end{pmatrix}=-\frac{1}{6}\begin{pmatrix} -6 \\ 18 \\ 6 \end{pmatrix}=\begin{pmatrix} 1 \\ -3 \\ -1 \end{pmatrix}.$$

So, the solutions of the system are as follows: $x_1=1$, $x_2=-3$, $x_3=-1$.

2) Let's solve this system by the Gauss method. As is known, by the Gauss method, unknowns are alternately excluded and found. To do this, we create an extended matrix that includes the coefficients of the system and free terms, and using elementary transformations, we easily perform calculations.

$$C=\left(\begin{array}{ccc|c} 1 & 1 & 2 & -4 \\ 4 & 1 & 4 & -3 \\ 2 & -1 & 2 & 3 \end{array}\right)\sim\left(\begin{array}{ccc|c} 1 & 1 & 2 & -4 \\ 1 & 4 & 4 & -3 \\ -1 & 2 & 2 & 3 \end{array}\right)\sim\left(\begin{array}{ccc|c} 1 & 1 & 2 & -4 \\ 1 & 4 & 4 & -3 \\ 0 & 3 & 4 & -1 \end{array}\right)\sim$$

$$\sim\left(\begin{array}{ccc|c} 1 & 1 & 2 & -4 \\ 0 & 3 & 2 & 1 \\ 0 & 3 & 4 & -1 \end{array}\right)\sim\left(\begin{array}{ccc|c} 1 & 1 & 2 & -4 \\ 0 & -3 & -2 & -1 \\ 0 & 0 & 2 & -2 \end{array}\right)$$

Using the found matrix, we create a system in the form of a triangular matrix and alternately find the unknowns.

$$\begin{cases} x_1 + x_2 + 2x_3 = -4, \\ 3x_2 + 2x_3 = 1 \\ 2x_3 = -2 \end{cases} \Rightarrow \begin{cases} x_3 = -1, \\ x_2 + 2\cdot(-1) = 1, \\ x_1 + x_2 + 2\cdot(-1) = -4 \end{cases} \Rightarrow \begin{cases} x_3 = -1, \\ x_2 = -3, \\ x_1 - 3 - 2 = -4 \end{cases} \Rightarrow \begin{cases} x_1 = -1, \\ x_2 = -3, \\ x_3 = 1. \end{cases}$$

CONCLUSION AND SUGGESTIONS

From the above, the following conclusions can be drawn: when teaching the topic "Matrices and actions on them" in a higher mathematics course, it is advisable to start by answering the question "What is a matrix and why is it needed?", giving the tasks that lead to the concept of a matrix, and explaining the advantages of using matrices. Then theoretical materials can be provided [7,8,9,10].

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