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CALCULATION OF MATRIX MULTIPLES

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Abstract:

This article provides information that is useful to high school students, including calculating the characteristic equation of matrixes, finding their own numbers and vectors, methods of calculating matrix multiples, and the necessary formulas. Thisinformation can be used to calculate engineering issues.

Keywords: matrix, characteristic equation, unique number, specific vector, matrix multiplicity.

Introduction

Matrix multiples are important in linear algebra and mathematical analysis. They allow you to solve a variety of issues related to the equations and systems in the matrix view. Matrix multiples are used to model and analyze dynamic systems. They allow you to describe the behavior of the system over time and predict its future state.

Before calculating the matrix multiples, let's take a look at the question of finding the characteristic equation, unique number and vector of the matrix.

For a vector $\vec{x}$ different from zero

$$A\vec{x} = \lambda\vec{x} \quad (1)$$

If the equation is performed, then the λ number A of square matrixes is called the unique number. A vector different from any zero that satisfies this equation is $\vec{x}$ called a specific vector A that matches the specific number of matrix λ . (1) Parity can be written as follows:

$$A\vec{x} - \lambda\vec{x} = 0, (A - \lambda E)\vec{x} = 0 \quad (2)$$

$$det(A - \lambda E) = 0 \quad (3)$$

(3) Equality is A called the characteristic equation of the matrix. Here is a E unit matrix whose matrix size is the same as that of a matrix [1,2]. A

Example 1. $A = \begin{pmatrix} 3 & 2 \\ 1 & 2 \end{pmatrix}$ find the specific number and specific vector of the matrix.

Unraveling. $\begin{vmatrix} 3-\lambda & 2 \\ 1 & 2-\lambda \end{vmatrix} = 0$ –This is A is called the characteristic equation of the matrix. If we solve this equation,

$$(3-\lambda)(2-\lambda) - 2 = 0$$

$$6 - 3\lambda - 2\lambda + \lambda^2 - 2 = 0$$

$$\lambda^2 - 5\lambda + 4 = 0$$
$$\lambda_1 = 1, \lambda_2 = 4.$$

$P_n(\lambda) = \lambda^2 - 5\lambda + 4$ - called the unique abusiveness of the matrix.

A We put the specific numbers of the matrix (2) in equation:

Case 1. $\lambda_1 = 1$ Let it be.

$$\begin{pmatrix} 3-1 & 2 \\ 1 & 2-1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = 0$$
$$\begin{pmatrix} 2 & 2 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = 0$$
$$\begin{cases} 2x_1 + 2x_2 = 0 \\ x_1 + x_2 = 0 \end{cases}$$
$$x_1 = -x_2 . . \ x_2 = 1 \text{ And } . \ x_1 = -1$$

Therefore, a $\lambda_1 = 1$ vector unique to its number $\vec{x}(-1,1)$.

Case 2. $\lambda_2 = 4$ Let it be.

$$\begin{pmatrix} 3-4 & 2 \\ 1 & 2-4 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = 0$$
$$\begin{pmatrix} -1 & 2 \\ 1 & -2 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = 0$$
$$\begin{cases} -x_1 + 2x_2 = 0 \\ x_1 - 2x_2 = 0 \end{cases}$$
$$x_1 = 2x_2 . . \ x_2 = 1 \text{ And } . \ x_1 = 2$$

So the $\lambda_2 = 4$ specific vector for the specific number $\vec{x}(2,1)$.

A The whole positive levels of the matrix are determined as follows:

$$A^0 = E, , \ A^1 = A, . \ A^n = A^{n-1} \cdot A.$$

$D = T^{-1} \cdot A \cdot T$ – A The Moon is a Monster. T Matrix is called the transition matrix. Otherwise $A = T \cdot D \cdot T^{-1}$ It will be. A The following properties are appropriate for the levels of the matrix:

I. $A^2 = (T \cdot D \cdot T^{-1})(T \cdot D \cdot T^{-1}) = T \cdot D \cdot (T^{-1} \cdot T) \cdot D \cdot T^{-1} = TD^2T^{-1}$.

II. $A^n = TD^nT^{-1}$.

III. $A^n + A^m = TD^nT^{-1} + TD^mT^{-1} = T(D^n + D^m)T^{-1}$.

IV. $\alpha A^n + \beta A^m = \alpha T D^n T^{-1} + \beta T D^m T^{-1} = T\left(\alpha D^n + \beta D^m\right)T^{-1}$.

n - Be given a level multiplier:

$$P_n(x) = a_n x^n + a_{n-1}x^{n-1} + ... + a_1 x + a_0 \ .$$

A The following matrix is said to be the majority of the matrix:

$$P_n(A) = a_n A^n + a_{n-1}A^{n-1} + ... + a_1 A + a_0 E \ ,$$

Here the levels are all positive numbers, E — the unit matrix.

All algebraic actions identified for multiples have also been identified for matrix multiples. $P_n(\lambda)$ If the matrix is unique, then [$P_n(A) = 0$ 3].

Example 2. $A = \begin{pmatrix} 3 & 2 \\ 1 & 2 \end{pmatrix}$. $P_n(x) = 4x^2 + 4x + 1$ If so, $P_n(A)$ nor hisoblang.

Method 1. $P_n(A) = 4A^2 + 4A + 1 = 4\begin{pmatrix} 3 & 2 \\ 1 & 2 \end{pmatrix}^2 + 4\begin{pmatrix} 3 & 2 \\ 1 & 2 \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} =$

$$= 4\begin{pmatrix} 11 & 10 \\ 5 & 6 \end{pmatrix} + \begin{pmatrix} 12 & 8 \\ 4 & 8 \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} =$$

$$= \begin{pmatrix} 44 & 40 \\ 20 & 24 \end{pmatrix} + \begin{pmatrix} 12 & 8 \\ 4 & 8 \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 57 & 48 \\ 24 & 33 \end{pmatrix}.$$

Method 2. $P_n(A) = 4A^2 + 4A + 1 = (2A+1)^2 = \left(2\begin{pmatrix} 3 & 2 \\ 1 & 2 \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\right)^2 =$

$$= \left(\begin{pmatrix} 6 & 4 \\ 2 & 4 \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\right)^2 = \begin{pmatrix} 7 & 4 \\ 2 & 5 \end{pmatrix}^2 = \begin{pmatrix} 57 & 48 \\ 24 & 33 \end{pmatrix}.$$

Method 3. A we create a matrix that goes through the specific vectors of the matrix:

$$T = \begin{pmatrix} -1 & 2 \\ 1 & 1 \end{pmatrix}.$$

T matritsaga teskari matritsani topamiz: $T^{-1} = \begin{pmatrix} \frac{-1}{3} & \frac{2}{3} \\ \frac{1}{3} & \frac{1}{3} \end{pmatrix}$.

$D = T^{-1} \cdot A \cdot T$ from parity D We create a diagonal matrix:

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$$D=\begin{pmatrix}-\frac{1}{3} & \frac{2}{3}\\ \frac{1}{3} & \frac{1}{3}\end{pmatrix}\begin{pmatrix}3 & 2\\ 1 & 2\end{pmatrix}\begin{pmatrix}-1 & 2\\ 1 & 1\end{pmatrix}=\begin{pmatrix}-\frac{1}{3} & \frac{2}{3}\\ \frac{4}{3} & \frac{4}{3}\end{pmatrix}\begin{pmatrix}-1 & 2\\ 1 & 1\end{pmatrix}=\begin{pmatrix}1 & 0\\ 0 & 4\end{pmatrix}.$$

Therefore, the diagonal elements of the diagonal matrix A consisted of the numbers of matrixes. We use the following equation to calculate the matrix multiple:

$$P_n(A)=T\cdot f(D)\cdot T^{-1},$$

here

$$D=\begin{pmatrix}d_1 & 0\\ 0 & d_2\end{pmatrix},\ f(D)=\begin{pmatrix}P_n(d_1) & 0\\ 0 & P_n(d_2)\end{pmatrix}. \qquad (4)$$

$$P_n(A)=\begin{pmatrix}-1 & 2\\ 1 & 1\end{pmatrix}\begin{pmatrix}9 & 0\\ 0 & 81\end{pmatrix}\begin{pmatrix}-\frac{1}{3} & \frac{2}{3}\\ \frac{1}{3} & \frac{1}{3}\end{pmatrix}=\begin{pmatrix}-9 & 162\\ 9 & 81\end{pmatrix}\begin{pmatrix}-\frac{1}{3} & \frac{2}{3}\\ \frac{1}{3} & \frac{1}{3}\end{pmatrix}=\begin{pmatrix}57 & 48\\ 24 & 33\end{pmatrix}.$$

Example 3. Using the above calculations, $P_n(x)=2x^6+4x^4+x$ for multiples $P_n(A)$ Calculate. Here $A=\begin{pmatrix}3 & 2\\ 1 & 2\end{pmatrix}$.

Solve. $P_n(A)=\begin{pmatrix}-1 & 2\\ 1 & 1\end{pmatrix}\begin{pmatrix}7 & 0\\ 0 & 9220\end{pmatrix}\begin{pmatrix}-\frac{1}{3} & \frac{2}{3}\\ \frac{1}{3} & \frac{1}{3}\end{pmatrix}=$

$$=\begin{pmatrix}-7 & 18440\\ 7 & 9220\end{pmatrix}\begin{pmatrix}-\frac{1}{3} & \frac{2}{3}\\ \frac{1}{3} & \frac{1}{3}\end{pmatrix}=\begin{pmatrix}6149 & 6142\\ 3071 & 3078\end{pmatrix}.$$

Example 4. $P_n(x)=2x^5+4x^3+x^2$ Multiplied. $A=\begin{pmatrix}4 & 0 & 1\\ -2 & 1 & 0\\ -2 & 0 & 1\end{pmatrix}$ If so, $P_n(A)$ nor hisoblang.

Solve. $\begin{vmatrix}4-\lambda & 0 & 1\\ -2 & 1-\lambda & 0\\ -2 & 0 & 1-\lambda\end{vmatrix}=0$ – we create a characteristic equation.

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$$(4-\lambda)(1-\lambda)(1-\lambda)+2(1-\lambda)=0$$

$$(1-\lambda)\left(4-4\lambda-\lambda+\lambda^2+2\right)=0$$

$$\lambda_1=1,\ \lambda^2-5\lambda+6=0,,\ \lambda_2=2,,.\ \lambda_3=3.$$

De holda diagonal matritsa $D=\begin{pmatrix}1 & 0 & 0\\ 0 & 2 & 0\\ 0 & 0 & 3\end{pmatrix}$.

$\lambda_1=1$ For (2) we find a vector specific to the equation:

$$\begin{cases}3\cdot x_1+0\cdot x_2+x_3=0\\ -2\cdot x_1+0\cdot x_2+0\cdot x_3=0,\\ -2\cdot x_1+0\cdot x_2+0\cdot x_3=0\end{cases}\begin{pmatrix}0\\ x_2\\ 0\end{pmatrix}\Rightarrow\begin{pmatrix}0\\ 1\\ 0\end{pmatrix}.$$

$\lambda_2=2$ For (2) we find a vector specific to the equation:

$$\begin{cases}2\cdot x_1+0\cdot x_2+x_3=0\\ -2\cdot x_1-x_2+0\cdot x_3=0\\ -2\cdot x_1+0\cdot x_2-2\cdot x_3=0\end{cases},\ \begin{pmatrix}x_1=-x_3/2\\ x_1=-x_2/2\\ x_1=-x_3/2\end{pmatrix}\Rightarrow\begin{pmatrix}1\\ -2\\ -2\end{pmatrix}.$$

$\lambda_3=3$ For (2) we find a vector specific to the equation:

$$\begin{cases}x_1+0\cdot x_2+x_3=0\\ -2\cdot x_1-2\cdot x_2+0\cdot x_3=0,\\ -2\cdot x_1+0\cdot x_2-2\cdot x_3=0\end{cases}\begin{pmatrix}x_1=-x_3\\ x_1=-x_2\\ x_1=-x_3\end{pmatrix}\Rightarrow\begin{pmatrix}-1\\ 1\\ 1\end{pmatrix}.$$

We create a matrix for passing specific vectors: $T=\begin{pmatrix}0 & 1 & -1\\ 1 & -2 & 1\\ 0 & -2 & 1\end{pmatrix}$.

T matritsaga teskari matritsani topamiz: $T^{-1}=\begin{pmatrix}0 & 1 & -1\\ -1 & 0 & -1\\ -2 & 0 & -1\end{pmatrix}$.

$f(D)$ we calculate:

$$f(D)=\begin{pmatrix}7 & 0 & 0\\ 0 & 100 & 0\\ 0 & 0 & 603\end{pmatrix}$$

$$P_n(A)=T\cdot f(D)\cdot T^{-1}=\begin{pmatrix}0&1&-1\\1&-2&1\\0&-2&1\end{pmatrix}\begin{pmatrix}7&0&0\\0&100&0\\0&0&603\end{pmatrix}\begin{pmatrix}0&1&-1\\-1&0&-1\\-2&0&-1\end{pmatrix}=$$

$$=\begin{pmatrix}0&100&-603\\7&-200&603\\0&-200&603\end{pmatrix}\begin{pmatrix}0&1&-1\\-1&0&-1\\-2&0&-1\end{pmatrix}=\begin{pmatrix}1106&0&503\\-1006&7&-410\\-1006&0&-403\end{pmatrix}.$$

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