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FUZZY MODELING OF BIOTECHNOLOGICAL PROCESSES: A CASE STUDY OF ENRICHED KAOLIN PRODUCTION

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Abstract:
This article investigates the mathematical modeling of biotechnological processes based on fuzzy logic, using the technology of enriched kaolin production as an example. In industrial production, deep processing of natural mineral raw materials, improvement of their quality characteristics, and implementation of environmentally friendly technologies are of significant scientific and practical importance. Iron oxides and other mineral impurities contained in kaolin adversely affect its whiteness, chemical composition, and functional properties. The use of microorganisms for the biological leaching and removal of iron compounds is considered a promising approach to addressing this problem.
The complexity of the biological enrichment process, its dependence on multiple factors, nonlinear dynamics, and variability in the initial raw material composition make it difficult to describe the process with sufficient accuracy using conventional deterministic models. Therefore, this article proposes a mathematical model based on fuzzy set theory, linguistic variables, membership functions, and a fuzzy inference algorithm.

Keywords: biotechnological process, fuzzy modeling, kaolin, biological enrichment, kaolinite, fuzzy logic, Mamdani algorithm, mathematical model, microorganisms, iron oxides, optimization, intelligent control.

Introduction

Currently, the efficient use of natural mineral raw materials, improvement of their processing technologies, and production of high-quality products are among the important scientific and technical challenges in various industrial sectors. Improving the quality of kaolin, which is widely used in the ceramics, porcelain and earthenware, paper, paint, rubber, construction materials, and chemical industries, is a particularly relevant issue.
Kaolin mainly consists of the mineral kaolinite, whose ideal chemical formula is expressed as $Al_2Si_2O_5(OH)_4$. Naturally occurring kaolin deposits may contain iron oxides, titanium compounds, quartz, feldspar, organic substances, and other mineral impurities. These

impurities, particularly iron oxides, significantly affect the color, whiteness, and certain technological properties of kaolin.

Traditional kaolin beneficiation methods include mechanical separation, hydraulic classification, magnetic separation, flotation, and chemical treatment. However, the effectiveness of these methods depends on the mineralogical composition of the impurities, particle size, and the form in which iron occurs in the kaolin.

In recent years, increasing attention has been paid to the biological beneficiation of mineral raw materials, which involves using microorganisms to dissolve or remove undesirable impurities. During biological beneficiation, certain bacteria and fungi produce organic acids, complex-forming substances, and other metabolites as a result of their metabolic activity. These substances help increase the solubility of iron compounds.

The main complexity of this process is that the biological activity of microorganisms directly depends on environmental conditions. Changes in temperature, pH, oxygen concentration, nutrient concentration, microbial population, and process duration affect the quality of the final product. Therefore, it is important to account for uncertainty and variability when mathematically modeling the biological beneficiation process.

Fuzzy logic theory is an effective mathematical tool for addressing such problems. This approach makes it possible to describe complex processes that are difficult to represent using precise mathematical equations through linguistic rules.

The aim of this study is to develop a mathematical model of the biological process of enriched kaolin production based on fuzzy logic, determine the interactions between technological parameters, and develop an intelligent control model aimed at improving process efficiency.

The main objectives of the study are as follows:

- To investigate the physicochemical and biotechnological characteristics of the biological beneficiation of kaolin.
- To identify the main technological parameters affecting the process.
- To formulate input and output variables based on fuzzy set theory.
- To develop membership functions.
- To construct a fuzzy inference system based on the Mamdani algorithm.
- To develop theoretical foundations for evaluating biological beneficiation efficiency and process optimization.

Kaolin is an important industrial mineral found in sedimentary and residual deposits of aluminosilicate minerals. Its main constituent is kaolinite. To obtain high-quality products, foreign impurities in the mineral composition must be reduced.

The main parameters determining kaolin quality include:

- Al_2O_3 and SiO_2 content;
- Content of coloring oxides such as Fe_2O_3 and TiO_2;
- Whiteness;
- Particle-size distribution;
- Degree of dispersion;

- Moisture content;
- Mineralogical composition.

The biological beneficiation of kaolin is based on the use of the metabolic activity of microorganisms. In this process, metabolites produced by microorganisms facilitate the conversion of iron compounds into soluble forms.

The effectiveness of microorganisms used in biological beneficiation depends on their physiological characteristics, the composition of the medium, and technological conditions. Certain bacteria and fungi are capable of producing organic acids, which may influence the dissolution of iron oxides.

An important advantage of biological beneficiation is the possibility of reducing the consumption of certain chemical reagents and purifying mineral raw materials under relatively mild conditions. At the same time, the relatively long duration of biological processes, the sensitivity of microorganisms to environmental factors, and process variability are the main limitations of this technology.

Therefore, a mathematical model that accounts for the interactions between technological parameters is required to improve the efficiency of biological beneficiation.

In conventional mathematical modeling, technological processes are generally described using precise equations, differential equations, and algebraic relationships. However, in biological systems, it is difficult to express all relationships mathematically with precision due to variations in process parameters, individual characteristics of microorganisms, and environmental influences.

Fuzzy logic theory was developed by L. A. Zadeh in 1965 and provides opportunities for decision-making under uncertainty and modeling complex systems.

Research Methods

The mathematical representation of a fuzzy set is expressed as follows:

$A=\{(x,\mu A(x))|x\in X\}A=\{(x,\mu_A(x))\mid x\in X\}$

Where:

- A – fuzzy set;
- X – the domain of the variable under consideration;
- x – a specific value of the variable;
- μA(x) – the degree of membership of x in set A.

The membership function satisfies the following condition:

$0\leq\mu A(x)\leq10\leq\mu_A(x)\leq1$

If the membership degree is equal to 0, the element does not belong to the given fuzzy set. If it is equal to 1, the element fully belongs to the set. Intermediate values represent partial membership.

In the biological beneficiation process, for example, the temperature of the medium can be expressed not simply as an exact numerical value but through linguistic concepts such as "low", "medium", and "high".

When developing a fuzzy model of enriched kaolin production, the most important technological parameters are selected.

Input variables of the model:

- x_1 – pH value of the medium;
- x_2 – temperature, °C;
- x_3 – microorganism concentration;
- x_4 – biological treatment duration, hours;
- x_5 – iron oxide content in the initial raw material, %.

Output variables:

- y_1 – iron removal efficiency, %;
- y_2 – whiteness of enriched kaolin, %.

Thus, the general mathematical model can be expressed as follows:

Y=F(x1,x2,x3,x4,x5)Y=F(x_1,x_2,x_3,x_4,x_5)

Here, F is a nonlinear function constructed on the basis of a fuzzy inference system.

The main purpose of this model is to evaluate the results of biological beneficiation under different combinations of technological parameters.

When constructing the fuzzy mathematical model, three linguistic values are assigned to each input parameter: low, medium, and high. These values are selected based on the specified parameter ranges.

Table 2. Linguistic variables of the input parameters of the fuzzy mathematical model

No.	Parameter	Designation	Low	Medium	High
1	Medium pH value	x_1	2–4	3.5–5.5	5–7
2	Temperature (°C)	x_2	20–27	25–35	33–40
3	Microorganism concentration (g/L)	x_3	0.1–1.5	1–3.5	3–5
4	Process duration (hours)	x_4	12–48	36–84	72–120
5	Initial Fe_2O_3 content (%)	x_5	0.1–1	0.7–2	1.7–3

Mathematical Representation of Linguistic Variables

Each input parameter is represented by the following linguistic set:

Xi={Low, Medium, High}X_i=\{Low,\ Medium,\ High\}

Where:

- Low – represents the lower range of parameter values;
- Medium – represents the intermediate range of parameter values;
- High – represents the upper range of parameter values.

Triangular and trapezoidal membership functions are proposed to mathematically represent the linguistic values of each parameter in the fuzzy model.

The defined linguistic variables serve as the basis for the next stages of fuzzy mathematical modeling, namely the construction of membership functions and the formation of the fuzzy rule base.

Research Results

The Mamdani algorithm is considered appropriate for modeling the biological beneficiation process of enriched kaolin production.

The Mamdani method makes it possible to describe complex systems using linguistic rules.

Fuzzy inference is performed through the following stages:

1. Fuzzification of input parameters.
2. Activation of fuzzy rules.
3. Aggregation of rule outputs.
4. Defuzzification of the final result.

The model's rule base is constructed in the following form:

Rule 1: IF the pH value of the medium is optimal AND the temperature is medium AND the microorganism concentration is sufficient, THEN the iron removal efficiency is high.

Rule 2: IF the temperature is low AND the process duration is short, THEN the iron removal efficiency is low.

Rule 3: IF the microorganism concentration is sufficient AND the process duration is long AND the pH value of the medium is optimal, THEN the biological beneficiation efficiency is high.

Rule 4: IF the initial iron oxide content is high AND the iron removal efficiency is low, THEN the whiteness of the final product may be low.

Rule 5: IF the iron removal efficiency is high AND the mineral structure is preserved, THEN the quality characteristics of kaolin are high.

When developing the rule base, it is important to consider microbiological and mineralogical characteristics. For example, the biological removal potential of iron present as free oxides differs from that of iron incorporated into the kaolinite crystal lattice.

In the Mamdani algorithm, the activation degree of a rule is determined by the following expression:

αi=min⁡(μA1(x1),μA2(x2),...,μAn(xn))\alpha_i=\min(\mu_{A_1}(x_1),\mu_{A_2}(x_2),..., \mu_{A_n}(x_n))

Where α_i is the activation degree of the i-th rule, and μA is the membership function.

During defuzzification, the centroid method can be used:

y∗=∫y·μB(y) dy∫μB(y) dyy^*=\frac{\int y\cdot\mu_B(y)\,dy} {\int\mu_B(y)\,dy}

Where:

- y* – final crisp output value;
- μB(y) – membership function of the output fuzzy set;
- y – output parameter value.

This expression makes it possible to evaluate the biological beneficiation result numerically.

When evaluating the efficiency of biological beneficiation, it is important to compare the initial and final concentrations of iron compounds.

The iron removal efficiency is calculated using the following formula:

ηFe=C0−C1C0·100%\eta_{Fe}=\frac{C_0-C_1}{C_0}\cdot100\%

Where:

- ηFe – iron removal efficiency, %;
- C_0 – initial iron oxide content in the raw material;
- C_1 – iron oxide content in the product after biological treatment.

Kaolin whiteness is determined using specialized optical measuring instruments. Its value is related to the light reflectance properties of the material, and a reduction in iron oxide content may contribute to an increase in whiteness.

However, whiteness does not depend solely on iron content. Titanium compounds, organic impurities, particle size, and other mineralogical characteristics also influence the final result. Therefore, considering iron removal efficiency and whiteness as separate output indicators in the model is a scientifically appropriate approach.

The analysis shows that the biological beneficiation of kaolin occurs under the simultaneous influence of biological, chemical, and physical factors.

Although conventional mathematical models can describe certain aspects of the process, they have specific limitations in accounting for the variability and uncertainty of biological systems. The main advantage of fuzzy logic-based modeling is that it enables existing expert knowledge about technological processes to be transformed into mathematical rules.

Conclusion

This article theoretically analyzes the mathematical modeling of biotechnological processes using fuzzy logic, taking the biological beneficiation of enriched kaolin production as an example. The following conclusions have been drawn:

1. The biological beneficiation of kaolin is a promising technological approach aimed at reducing iron compounds in natural mineral raw materials and improving the quality characteristics of the final product.
2. The efficiency of biological beneficiation directly depends on the pH value of the medium, temperature, microorganism concentration, process duration, and initial raw material composition.
3. The nonlinear and variable characteristics of biological processes require the use of fuzzy logic theory alongside conventional deterministic modeling approaches.
4. Within the scope of this study, a conceptual fuzzy mathematical model structure was proposed, taking into account pH, temperature, microorganism concentration, process duration, and initial iron content.
5. Linguistic rules developed on the basis of the Mamdani algorithm make it possible to evaluate the interactions between technological parameters and predict the results of biological beneficiation.

In general, fuzzy logic-based modeling is one of the promising scientific approaches to the intelligent control of biological technological processes for enriched kaolin production, optimization of technological operating conditions, and production of high-quality mineral products.

References

1. Zadeh, L. A. (1965). Fuzzy Sets. Information and Control, 8(3), 338–353. https://doi.org/10.1016/S0019-9958(65)90241-X.
2. Mamdani, E. H., & Assilian, S. (1975). An Experiment in Linguistic Synthesis with a Fuzzy Logic Controller. International Journal of Man-Machine Studies, 7(1), 1–13. https://doi.org/10.1016/S0020-7373(75)80002-1.
3. Shakirov, Sh. Yu., Salimov, Z. S., Shakirov, T. I., & Mkrtchyan, R. V. (1993). Enrichment of Angren Kaolins Using an Electromagnetic Separator. Uzbek Chemical Journal, No. 1, pp. 52–58. (In Russian).
4. Kafarov, V. V. (1985). Methods of Cybernetics in Chemistry and Chemical Technology. Moscow: Khimiya, 448 p. (In Russian).
5. Kariman, S. A., Braitsev, A. V., et al. Modeling and Optimization of Production Processes in Coal Mining. (Additional bibliographic details are required).
6. Averkin, A. N., et al. (1986). Fuzzy Sets in Artificial Intelligence Control Models. Moscow: Nauka. (In Russian).
7. Kamolov, E. R. (2020). Development of an Algorithm for Optimizing the Technological Process of Kaolin Beneficiation. Science and Society Journal of Uzbekistan, No. 1, pp. 10–14. (Bibliographic details translated from the original title).
8. Komolov, E. R. (2020). Decision Development of Management Problems of Biotechnological Systems under Uncertainty of Environmental States Using Mathematical Statistics Methods. International Scientific Journal, No. 3.
9. Kamolov, E. R. Development of an Algorithm for Optimizing the Technological Process of Kaolin Enrichment. In: 2020 IEEE International IOT, Electronics and Mechatronics Conference (IEMTRONICS).